7.5 Congruent Triangles to the Rescue

A Practice Understanding Task

Part 1

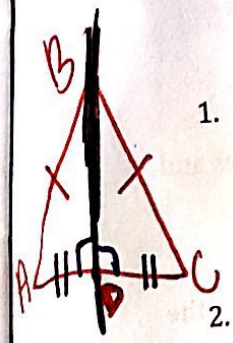
Zac and Sione are exploring isosceles triangles—triangles in which two sides are congruent:

Zac: I think every isosceles triangle has a line of symmetry that passes through the vertex point of the angle made up by the two congruent sides, and the midpoint of the third side.

Sione: That's a pretty big claim—to say you know something about *every* isosceles triangle. Maybe you just haven't thought about the ones for which it isn't true.

Zac: But I've folded lots of isosceles triangles in half, and it always seems to work.

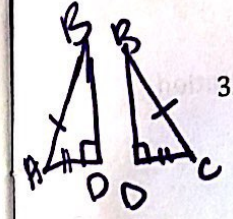
Sione: *Lots* of isosceles triangles are not *all* isosceles triangles, so I'm still not sure.



1. What do you think about Zac's claim? Do you think every isosceles triangle has a line of symmetry? If so, what convinces you this is true? If not, what concerns do you have about his statement? yes . . . No . . . .

2. What else would Zac need to know about the crease line through in order to know that it is a line of symmetry? (Hint: Think about the definition of a line of reflection.) Perpendicular Bisector

Pictures in Notebook

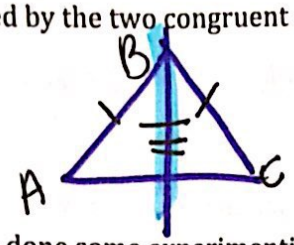


3. Sione thinks Zac's "crease line" (the line formed by folding the isosceles triangle in half) creates two congruent triangles inside the isosceles triangle. Which criteria—ASA, SAS or SSS—could he use to support this claim? Describe the sides and/or angles you think are congruent, and explain how you know they are congruent. ASA, SSS, SAS all work!

HL *Donkey came up, but we have a right angle!

4. If the two triangles created by folding an isosceles triangle in half are congruent, what does that imply about the "base angles" of an isosceles triangle (the two angles that are not formed by the two congruent sides)? in Notebook! CPCTC

5. If the two triangles created by folding an isosceles triangle in half are congruent, what does that imply about the "crease line"? (You might be able to make a couple of claims about this line—one claim comes from focusing on the line where it meets the third, non-congruent side of the triangle; a second claim comes from focusing on where the line intersects the vertex angle formed by the two congruent sides.)



* Reflexive property therefore
* share the side, ∴ sides are ≅

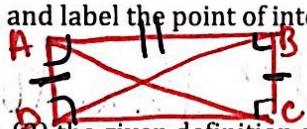
Part 2

Like Zac, you have done some experimenting with lines of symmetry, as well as rotational symmetry. In the tasks *Symmetries of Quadrilaterals* and *Quadrilaterals—Beyond Definition* you made some observations about sides, angles, and diagonals of various types of quadrilaterals based on your experiments and knowledge about transformations. Many of these observations can be further justified based on looking for congruent triangles and their corresponding parts, just as Zac and Sione did in their work with isosceles triangles.

Pick one of the following quadrilaterals to explore:

- A **rectangle** is a quadrilateral that contains four right angles.
- A **rhombus** is a quadrilateral in which all sides are congruent.
- A **square** is both a rectangle and a rhombus, that is, it contains four right angles and all sides are congruent

1. Draw an example of your selected quadrilateral, with its diagonals. Label the vertices of the quadrilateral *A*, *B*, *C*, and *D*, and label the point of intersection of the two diagonals as point *N*.



2. Based on (1) your drawing, (2) the given definition of your quadrilateral, and (3) information about sides and angles that you can gather based on lines of reflection and rotational symmetry, list as many pairs of congruent triangles as you can find.

∠A, ∠B, ∠C, ∠D are 90° ∠s, ∴ ≅
$\overline{AD} \cong \overline{BC}$, $\overline{AB} \cong \overline{DC}$ (opp sides ≅)

3. For each pair of congruent triangles you list, state the criteria you used—ASA, SAS or SSS—to determine that the two triangles are congruent, and explain how you know that the angles and/or sides required by the criteria are congruent (see the following chart).

Congruent Triangles	Criteria Used (ASA, SAS, SSS)	How I know the sides and/or angles required by the criteria are congruent
If I say $\Delta RST \cong \Delta XYZ$	based on SSS	then I need to explain: • how I know that $\overline{RS} \cong \overline{XY}$, and • how I know that $\overline{ST} \cong \overline{YZ}$, and • how I know that $\overline{TR} \cong \overline{ZX}$ so I can use SSS criteria to say $\Delta RST \cong \Delta XYZ$
$\Delta ABC \cong \Delta CDA$	SAS	$\overline{AD} \cong \overline{BC}$ → opp sides ≅ $\angle D \cong \angle B$ → Rec have 90° ∠s $\overline{DC} \cong \overline{AB}$ → opp sides ≅
$\Delta ABC \cong \Delta CDA$	HL	$\angle D \cong \angle B$ → Rec have 90° ∠s $\overline{AC} \cong \overline{AC}$ → Reflexive Property $\overline{AD} \cong \overline{BC}$ → opp sides ≅
$\Delta ABC \cong \Delta DCB$ Δ [illegible]	SAS	[illegible] → opp sides ≅ $\overline{AB} \cong \overline{DC}$ → Rec has 90° ∠s $\angle C \cong \angle B$ $\overline{BC} \cong \overline{BC}$ → Reflexive Property

HW

4. Now that you have identified some congruent triangles in your diagram, can you use the congruent triangles to justify something else about the quadrilateral, such as:

   - the diagonals bisect each other
   - the diagonals are congruent
   - the diagonals are perpendicular to each other
   - the diagonals bisect the angles of the quadrilateral

   Pick one of the bulleted statements you think is true about your quadrilateral and try to write an argument that would convince Zac and Sione that the statement is true.

After working with these equations and seeing the transformations on the coordinate graph it is good timing to consider similar work with tables.

6. Match the table of values below with the proper function rule.

I

x	f(x)
-1	16
0	14
1	12
2	10

II

x	f(x)
-1	14
0	12
1	10
2	8

III

x	f(x)
-1	12
0	10
1	8
2	6

IV

x	f(x)
-1	10
0	8
1	6
2	4

V

x	f(x)
-1	8
0	6
1	4
2	2

A. $f(x) = -2(x-1)+8$

B. $f(x) = -2(x-1)+12$

C. $f(x) = -2(x-2)+8$

D. $f(x) = -2(x+1)+8$

E. $f(x) = -2(x+1)+10$

SET

Topic: Use Triangle Congruence Criteria to justify conjectures.

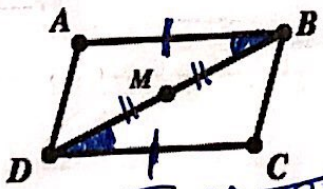
In each problem below there are some true statements listed. From these statements a conjecture (a guess) about what might be true has been made. Using the given statements and conjecture statement create an argument that justifies the conjecture.

7. True statements:

Point M is the midpoint of $\overline{DB}$

$\angle ABD \cong \angle BDC$

$\overline{AB} \cong \overline{DC}$



Conjecture: $\angle A \cong \angle C$

a. Is the conjecture correct?

yes

b. Argument to prove you are right:

$\overline{BD} \cong \overline{BD}$ Reflective Property

$\triangle ABD \cong \triangle CDB$, by SAS

$\angle A \cong \angle C$, CPCTC

① Look at the true statements "given"

② Mark given on my △

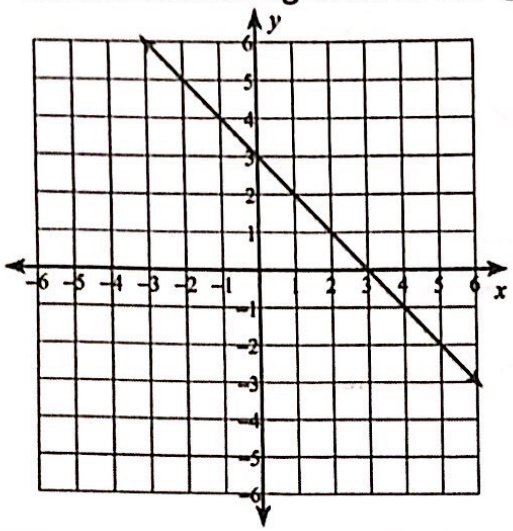
③ Prove △s ≅

④ Use CPCTC

GO
Topic: Average Rate of Change

Slope

11. What is the average rate of change of $f(x)$ over the interval $-2 < x < 3$



rise / run

12. What is the average rate of change of $f(x)$ over the interval $-4 < x < 2$

x	$f(x)$ "y"
-4	-4
-3	-3.5
-2	-3
-1	-2.5
0	-2
1	-1.5
2	-1
3	-0.5
4	0

$(-4, -4)$
(x_1, y_1)

$(2, -1)$
(x_2, y_2)

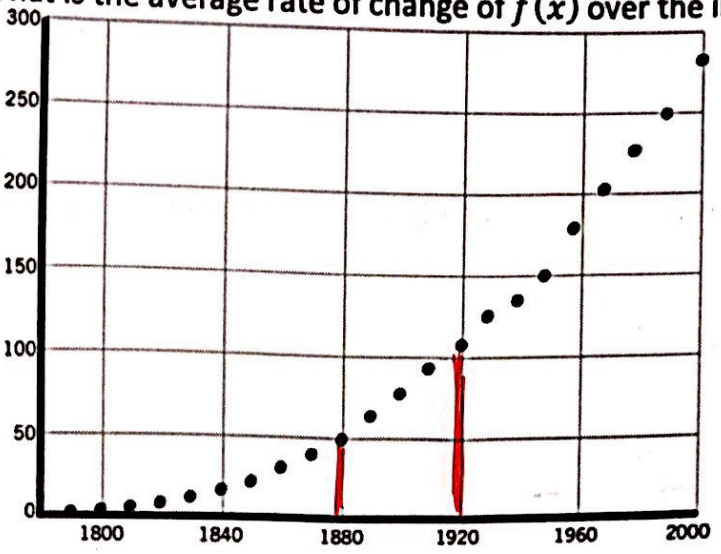
change
$\frac{\Delta y}{\Delta x}$

$= \frac{y_2 - y_1}{x_2 - x_1}$

$y_2 = -1$

$= (-1) -$

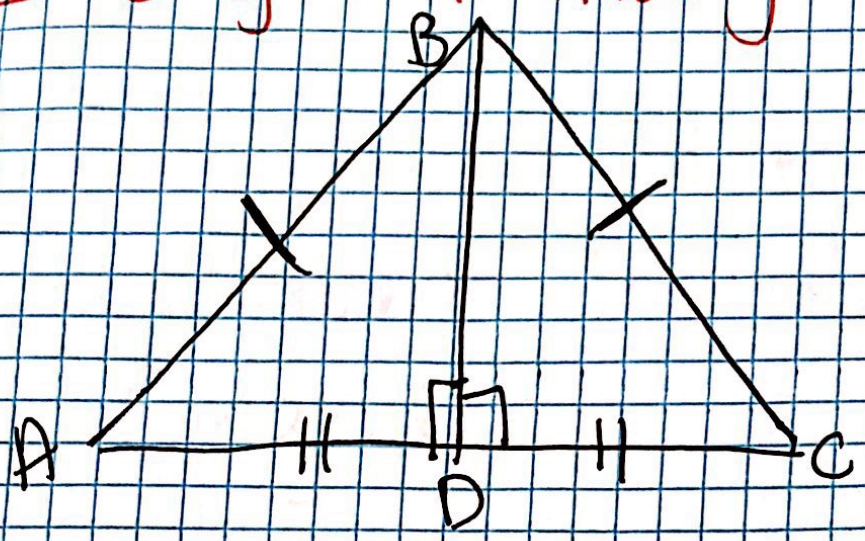
13. What is the average rate of change of $f(x)$ over the interval $1880 < x < 1920$



14. What is the average rate of change of $f(x)$ over the interval $1810 < x < 1910$

x	$f(x)$
1790	3.9
1800	5.3
1810	7.2
1820	9.6
1830	12.9
1840	17.1
1850	23.2
1860	31.4
1870	39.8
1880	50.2
1890	63.0
1900	76.2
1910	92.2
1920	106.0
1930	123.2
1940	132.2
1950	151.3
1960	179/3
1970	203.3
1980	226.5
1990	248.7
2000	281.4

1.5 Congruent Triangles



★ Congruent by HL

$\triangle ABD \cong \triangle CBD$

$\angle A \cong \angle C$ $\quad \overline{BA} \cong \overline{BC}$

How did we know?

Order they appear in the congruency statement

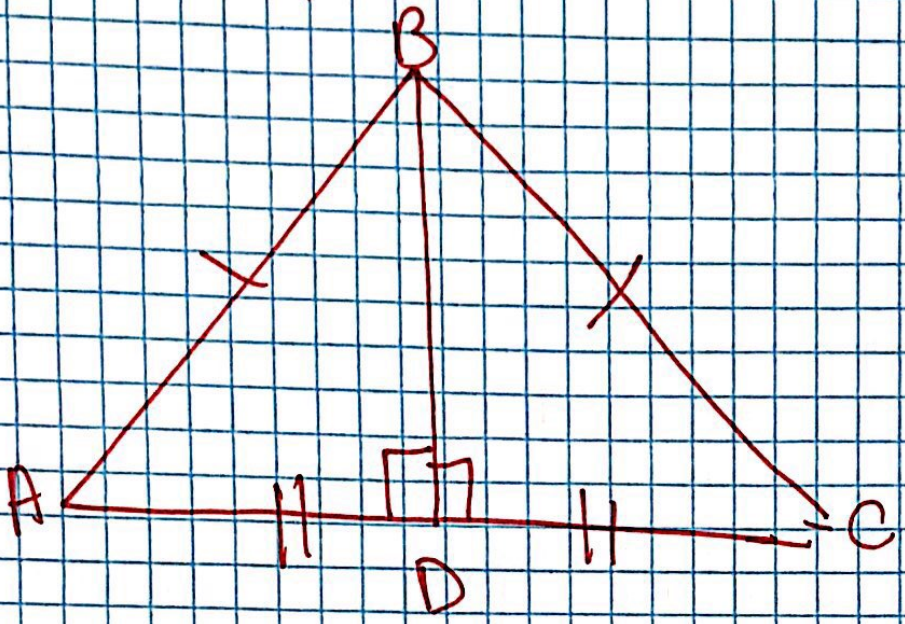
CPCTC -

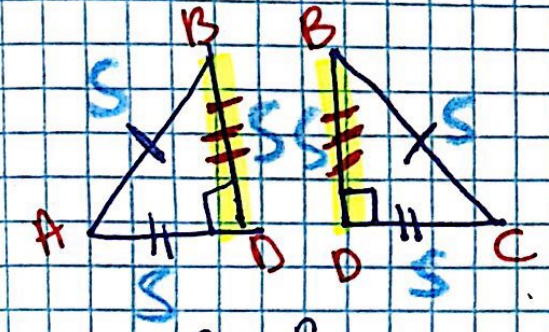
Corresponding Parts of Congruent Triangles are Congruent.

* If triangles are congruent, then all corresponding sides are congruent

[They match based off of congruency statement.]

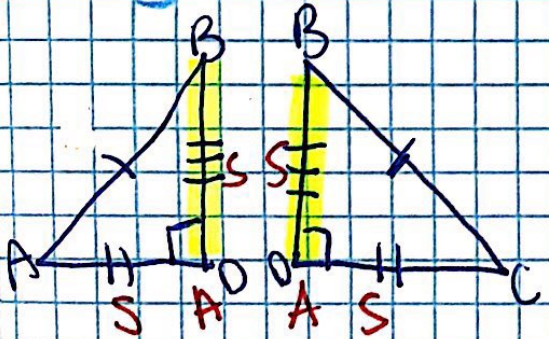
Looking into different Criteria using our Isosceles Triangle Example From 7.5





* Reflexive Property allows us to say $\overline{BD} \cong \overline{BD}$

We can then find SSS & SAS



If we remember base angles are congruent,..

We can then use ASA.