Unit 5 – Solving Quadratic Functions

Now that we know how to work with quadratics, we can solve them! What does it mean when we solve a quadratic? What are we looking for?

X-intercepts

Since we have quadratics we should be looking for two solutions!

We can write our solutions as ordered pairs. (#, 0) (#,0) or as X=# or X=#.

Solving Methods:
- -Factoring/Zero Product Property
- -Square Root Method
- -Quadratic Formula

Today we will talk through helping us find the best strategy depending on the quadratics we are given.

Part 1- Factoring (Zero Product Property) Method: If quadratics are factored, or if we can feel confident in our ability to factor a given problem, this method can be super easy!

All we need to do is set equal to zero and solve for x!

1. (x – 2) (x + 4)

$\boxed{x=2 \quad x=-4}$

2. (2x – 3)(x – 5)

$\boxed{x=\frac{3}{2} \quad x=5}$

3. x (3x +2)

$\boxed{x=0 \quad x=-2/3}$

4. (x +12) (x – 6)

$\boxed{x=-12 \quad x=6}$

5. (x – 7) (2x – 5)

$\boxed{x=7 \quad x=5/2}$

6. x(x+9)

$\boxed{x=0 \quad x=-9}$

7. (x+5)(x – 15)

$\boxed{x=-5 \quad x=15}$

8. (x – 3)(x – 8)

$\boxed{x=3 \quad x=8}$

9. (x+6)(x+3)

$\boxed{x=-6 \quad x=-3}$

order to solve using the remaining methods we need to recall:

<u>Simplifying Radicals:</u>
When simplifying radicals it is important that we make a factor tree.

We want to look for groups of 2, → take out of radical

Anything left stays under the radical.

If multiple numbers are present in either location we must multiply.

Remember, negatives under the radical mean that we have imaginary solutions.

Simplify the following radicals completely.

1. $\sqrt{250}$ 250, 25 10, 5 5 5 2, $5\sqrt{5\cdot 2}$ $\boxed{5\sqrt{10}}$

2. $\sqrt{48}$ 48, 2 24, 2 12, 4 3, 2 2, $2\cdot 2\sqrt{3}$ $\boxed{4\sqrt{3}}$

3. $\sqrt{-16}$ 16, 4 4 $\boxed{4i}$

4. $\sqrt{216}$ 216, 2 108, 2 54, 2 27, 3 9, 3 3, $3\cdot 2\sqrt{2\cdot 3}$ $\boxed{6\sqrt{6}}$

5. $\sqrt{45}$ 45, 5 9, 3 3 $3\sqrt{5}$

6. $\sqrt{-98}$ −98, 2 49, 7 7 $\boxed{7i\sqrt{2}}$

7. $\sqrt{-169}$ 169, 13 13 $\boxed{13i}$

8. $\sqrt{200}$ 200, 2 100, 10 10 $\boxed{10\sqrt{2}}$

<u>Part 2- Square Root Method:</u> If quadratics are in vertex form, (or that we find confident putting it in this form) or a quadratic is missing the b-value this is the best method!

1. $(x+3)^2+2=-10$
$-2 \quad -2$
$(x+3)^2=-12$
$x+3=\sqrt{-12}$
$x=\sqrt{-12}-3$
$\boxed{x=\pm 2i\sqrt{3}-3}$

2. $2(x+2)^2+24=0$
$2(x+2)^2=-24$
$(x+2)^2=-12$
$x+2=\sqrt{-12}$
$x=\sqrt{-12}-2$
$\boxed{x=\pm 2i\sqrt{3}-2}$

3. $5x^2 - 1 = 9$

$5x^2 = 10$

$x^2 = 2$

$\boxed{x = \pm\sqrt{2}}$

4. $(x-4)^2 - 13 = 0$

$(x-4)^2 = 13$

$x - 4 = \sqrt{13}$

$\boxed{x = \pm\sqrt{13} + 4}$

5. $5 - 3x^2 = 20$

$-3x^2 = 15$

$x^2 = -5$

$x = \sqrt{-5}$

$\boxed{x = \pm i\sqrt{5}}$

6. $x^2 - 49 = 0$

$x^2 = 49$

$\boxed{x = \pm 7}$

7. $x^2 + 100 = 0$

$x^2 = -100$

$x = \sqrt{-100}$

$\boxed{x = \pm 10i}$

8. $2(x+3)^2 + 12 = 4$

$2(x+3)^2 = -8$

$(x+3)^2 = -4$

$x + 3 = \sqrt{-4}$

$x = \sqrt{-4} - 3$

$\boxed{x \pm 2i - 3}$

9. $5x^2 + 9 = 134$

$5x^2 = 125$

$x^2 = 25$

$\boxed{x = \pm 5}$

10. $2x^2 + 8 = 10$

$2x^2 = 2$

$x^2 = 1$

$\boxed{x = \pm 1}$

Part 3- The Quadratic Formula: If quadratics have an a-value that is bigger than one, or we do not know how to approach in solving, the quadratic formula can always work!

State the Quadratic Formula:

To help you remember, you could sing our song… or think about this story! *"There once was a negative boy who was unsure if he wanted to go to a radical party. But the boy was feeling squared and missed out on four awesome chicks! The party was not over until 2 am!*

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solving using the Quadratic Formula:

1. $x^2 + 16x + 68 = 0$

a= 1 b= 16 c= 68

$$X = \frac{-16 \pm \sqrt{(16)^2 - 4(1)(68)}}{2(1)}$$

$$X = \frac{-16 \pm \sqrt{-16}}{2} = \frac{-16 \pm 4i}{2} = \boxed{-8 \pm 2i}$$

2. $2x^2 - 7x + 6 = 0$

a= 2 b= -7 c= 6

$$X = \frac{7 \pm \sqrt{(-7)^2 - 4(2)(6)}}{2(2)}$$

$$X = \frac{7 \pm \sqrt{1}}{4} = \frac{7 \pm 1}{4}$$

$$X = \frac{8}{4} = 2 \qquad X = \frac{6}{4} = \frac{3}{2}$$

3. $x^2 - 8x + 24 = 0$

a= 1 b= -8 c= 24

-32
2 16
4 4

$$X = \frac{8 \pm \sqrt{(-8)^2 - 4(1)(24)}}{2(1)}$$

$$X = \frac{8 \pm \sqrt{-32}}{2} = \frac{8 \pm 4i\sqrt{2}}{2} =$$

$$\boxed{x = 4 \pm 2i}$$

4. $2x^2 - 32 = 0$

a= 2 b= 0 c= -32

$$X = \frac{0 \pm \sqrt{0^2 - 4(2)(-32)}}{2(2)}$$

$$X = \frac{0 \pm \sqrt{256}}{4} = \frac{0 \pm 16}{4}$$

$$x = \frac{16}{4} = 4 \qquad x = \frac{-16}{4} = -4$$

5. $6x^2 + 2x + 1 = 0$

a= 6 b= 2 c= 1

-20
2 10
2 5

$$X = \frac{-2 \pm \sqrt{(2)^2 - 4(6)(1)}}{2(6)}$$

$$X = \frac{-2 \pm \sqrt{-20}}{12} = \frac{-2 \pm 2i\sqrt{5}}{12} = \boxed{\frac{-1 \pm i\sqrt{5}}{6}}$$

6. $x^2 + 5x - 6 = 0$

a= 1 b= 5 c= -6

$$X = \frac{-5 \pm \sqrt{(5)^2 - 4(1)(-6)}}{2(1)}$$

$$X = \frac{-5 \pm \sqrt{49}}{2} = \frac{-5 \pm 7}{2}$$

$$x = \frac{-12}{6} = -2 \qquad x = \frac{2}{2} = 1$$

Calculator Strategies:

- We can also try to use our calculator while testing to help us find solutions!
  - Use your table! We want to look for when $y=0$!
    - Careful: Not all answers can be seen from the table.

x	y
-2	0
-1	-4
0	-6
1	-6
2	-4
3	0

Example: $x^2 - x - 6$

Solutions: $x=-2$ or $x=3$

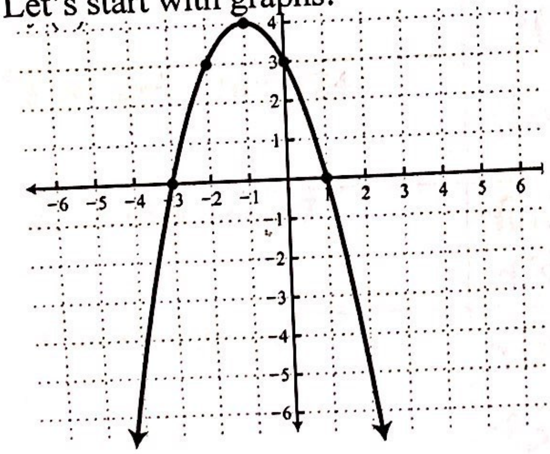
  - Trace Feature: We can trace in our calculators to find where the x-intercepts are. We can then use those values to match- up our answer choices.
    - Steps:
      - $2^{nd} \rightarrow$ Trace
      - Zero
      - Left Bound, Right Bound, Enter!

Example: $3x^2 - 5x - 15$

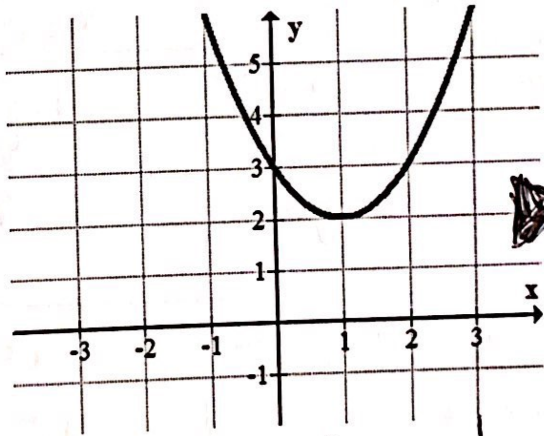
Solutions: $x=-1.55$ or $x=3.22$

Types of solutions: We can look at our graph as well as the descriminant of a quadratic to tell us the types of solutions! Let's take a look to see how this works!

Let's start with graphs:

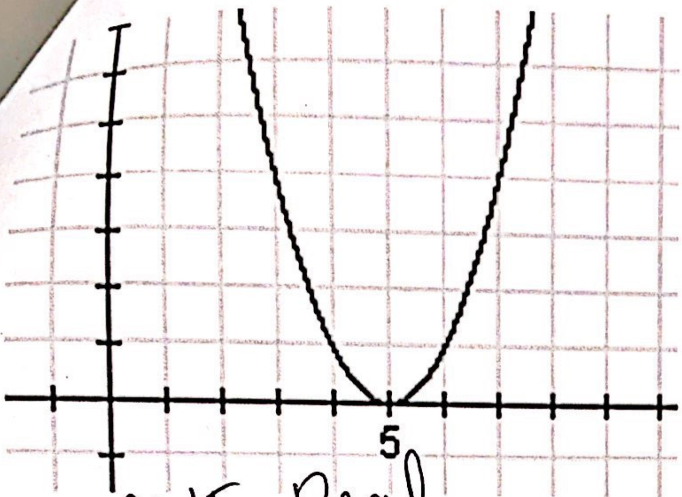


2 Solutions
Real
[Rational]

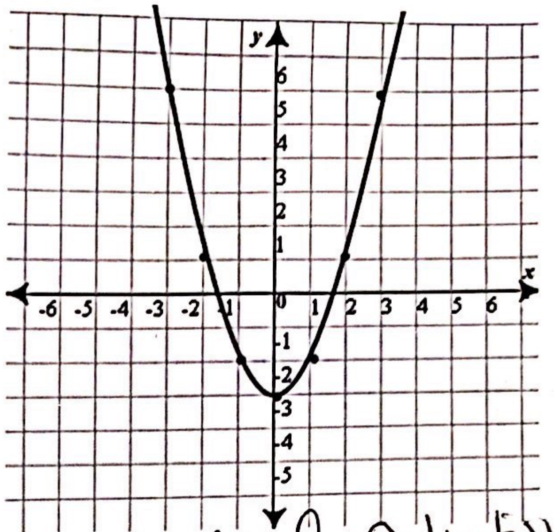


Does not cross x-axis

No real Solutions!
[Imaginary]
2 of them!



ONE Real Solution → Rational



TWO Real Solutions Irrational

Let's look at Discriminants! (Fancy way of using what we have in the quadratic formula!)

We will use: $\sqrt{b^2-4ac}$

	Discriminant	Type of Solution
1. y= $3x^2 - 3x + 2$	$\sqrt{(-3)^2-4(3)(2)}$ $\sqrt{-15}$	TWO Imaginary
2. y= $x^2 - 10x + 1$	$\sqrt{(-10)^2-4(1)(1)}$ $\sqrt{96} = 9.798$	TWO Irrational [Real]
3. y= $2x^2 - 16x + 24$	$\sqrt{(-16)^2-4(2)(24)}$ $\sqrt{64} = 8$	TWO Rational [Real]
4. y= $x^2 + 4x + 17$	$\sqrt{(4)^2-4(1)(17)}$ $\sqrt{-52}$	IMAGINARY TWO
5. y= $x^2 + 2x - 6$	$\sqrt{(2)^2-4(1)(-6)}$ $\sqrt{28} \approx 5.29$	TWO Real Irrational
6. y= $2x^2 - x - 3$	$\sqrt{(-1)^2-4(2)(-3)}$ $\sqrt{25} = 5$	TWO Real Rational

☆ Rational → can be written as a fraction

☆ Irrational → cannot be written as a fraction